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ABSTRACTS

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ISDE

Institute of Mathematics and Computer Science
University of Latvia

DAUGAVPILS UNIVERSITY

Plenary Lectures

The fundamental bifurcation theorem for nonlinear matrix equations in the absence of strict dominance (and what does it have to do with cicadas?

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For a difference equation $x(t+1) = Px(t)$ with a nonnegative and irreducible matrix P, the equilibrium $x = 0$ losses stability as the dominant eigenvalue r of P increases through 1, at which point there is a continuum of positive equilibria. In a population dynamic setting, this fundamental bifurcation theorem concerns the basic question of extinction versus persistence of the modeled population. Most population models are, however, nonlinear. A fundamental bifurcation theorem for nonlinear matrix models $x(t+1) = P(x(t))x(t)$ asserts the bifurcation of positive equilibria as the inherent growth rate $r(0)$ (the dominant eigenvalue of $P(0)$) increases through 1. The spectrum of $r(0)$ values associated with the bifurcating continuum is then an interval, instead of a point, and the stability of the positive equilibria near bifurcation depends on the direction of bifurcation. This fundamental theorem requires that $r(0)$ be a *strictly* dominant eigenvalue of $P(0)$. For a biologically important class of models, however, $r(0)$ is not strictly dominant. It turns out that this mathematical subtlety relates to a basic question in population dynamics concerning life history strategies for individuals in biological populations, namely, whether to reproduce once (semelparity) or more than once (iteroparity). I will survey what is known about the fundamental bifurcation theorem for semelparous matrix models (when $r(0)$ is not strictly dominant). There will be many open problems.

[1] J. M. Cushing, Nonlinear semelparous Leslie models, *Mathematical Biosciences and Engineering* **3**, 1 (2006), 17-36.

[2] J. M. Cushing, Three stage semelparous Leslie models, *Journal of Mathematical Biology* **59** (2009), 75-104.

Dynamic classification of Sierpinski curve Julia sets

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In this talk we consider the family of rational maps of the form $z^n + C/z^d$ where $n > 1$. When all the critical orbits escape to infinity, it is known that the Julia set is a Sierpinski curve, i.e., homeomorphic to the Sierpinski carpet. While all these sets are homeomorphic, they have very different dynamics depending upon the behavior of the critical orbits. We give a complete classification of these dynamical behaviors.

Joint work with Kevin Pilgrim.

Difference equations for evolutionary games

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I will present several discrete time dynamical systems (=difference equations) that arise in evolutionary game theory, population genetics and population dynamics.

[1] J. Hofbauer and K. Sigmund, Evolutionary Game Dynamics, *Bull. Amer. Math. Soc.* **40** (2003), 479-519.

[2] T. Nagylaki, J. Hofbauer and P. Brunovský, Convergence of multilocus systems under weak epistasis or weak selection, *J. Math. Biology* **38** (1999), 103-133.

[3] F. Hofbauer, J. Hofbauer, P. Raith and T. Steinberger, Intermingled basins in a two species system, *J. Math. Biology* **49** (2004), 293-309.

Periodic solutions of second order nonlinear difference equations : variational approach

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Since 2003, results about the existence and multiplicity of T-periodic solutions of T-periodic systems of difference equations of the form

$$\Delta\phi[\Delta u(n-1)] = \nabla_u F[n, u(n)] + h(n) \quad (n \in \mathbb{Z})$$

have been obtained using variational methods and critical point theory. In this equation, $\phi = \nabla\Phi$, with Φ strictly convex, is a homeomorphism from its domain of definition in $\mathbb{R}^N$ onto $\mathbb{R}^N$. The classical situations correspond to $\phi(v) = |v|^{p-2}v$ ($p > 1$).
Special emphasis is made upon recent results for the case where ϕ is a homeomorphism from some open ball of $\mathbb{R}^N$ onto $\mathbb{R}^N$. Various conditions upon F and h are given for the existence of T-periodic solutions. The approach combines variational inequalities and Brouwer degree.

Chimera states and ideal turbulence

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In nonlinear dynamics, currently considerable attention has been focussed on research in so-called chimeras — the spatio-temporal states in which coherence and incoherence coexist [1,2]. In the talk we analyze a link between chimera states and ideal turbulence.

[1] D. M. Abrams and S. H. Strogatz, Chimera States for Coupled Oscillators, *Physical Review Letters* 93, 174102 (2004).

[2] O. E. Omel'chenko, Yu. L. Maistrenko and P. A. Tass, Chimera States: The natural Link between Coherence and Incoherence, *Physical Review Letters* 100, 044105 (2008).

[3] A. N. Sharkovsky and S. A. Berezovsky, Phase Transitions in correct-incorrect Calculations for some Evolution Problems, *Intern. J. Bifurcation and Chaos* 13, 7 (2003), 1811-1821.

[4] A. N. Sharkovsky and E. Yu. Romanenko, Turbulence, ideal, *Encyclopedia of Nonlinear Science, Routledge, Taylor & Francis*, 2005, 955-957.

Abstracts of Contributed Talks

Linear difference equations and inversion of matrices

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Algorithms in the inversion of almost-triangular and banded matrices, based on linear differences equations, have profusely used. Compact representations for the elements of the inverses of almost-triangular (tridiagonal and Hessenberg) matrices are considered. The entries of the inverse matrices are given in terms of determinants of proper submatrices with the same structure. It gives simple proofs of such algorithms. In addition, we introduce linear recurrences for the straightforward computation of the inverses of triangular matrices.

It is a joint work with Mustapha Rachidi, Académie de Reims, France.

Topological invariants of time-periodic dynamical systems

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Let $I \subset \mathbb{R}$ be a compact interval. By a time-periodic nonautonomous dynamical system on I we mean a periodic sequence

$$F = \{f_n\}_{n=0}^{+\infty},$$

of continuous maps $f_n : I \to I$.
The relationship between topological entropy and periodic entropy of F is the main subject of this talk.

[1] S. Kolyada, M. Misiurewicz and L. Snoha, Topological entropy of nonautonomous piecewise monotone dynamical systems on the interval, *Fund. Math.* **160**, 2 (1999), 161-181.

[2] S. Kolyada and L. Snoha, Topological entropy of nonautonomous dynamical systems, *Random Comput. Dynam.* **4**, 2-3 (1996), 205-233.

On the some second-order rational difference equations problems

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In our talk we discuss about some second-order rational difference equations in form

$$x_{n+1} = \frac{\alpha + \beta x_n + \gamma x_{n-1}}{A + Bx_n + Cx_{n-1}},\ n = 0, 1, 2, \dots,$$

and some second-order quadratic rational difference equation in form

$$x_{n+1} = \frac{\alpha + \beta x_n x_{n-1} + \gamma x_{n-1}}{A + Bx_n x_{n-1} + Cx_{n-1}},\ n = 0, 1, 2, \dots,$$

which are specially research in [1] and [2]. For example, we investigate the rational difference equation

$$x_{n+1} = \frac{\alpha}{(1 + x_n)x_{n-1}},\ n = 0, 1, 2, \dots.$$

We give some answers of Open Problems whose are formulated in [1] and [2].

[1] A.M. Amleh, E. Camouzis and G. Ladas, On the Dynamics of a Rational Difference Equations, Part 1, *International Journal of Difference Equations* **3**, 1 (2008), 1–35.

[2] M.R.S. Kulenovič and G. Ladas, Dynamics of Second Order Rational Difference Equations. With Open Problems and Conjectures, *Chapman&Hall/CRC, USA*, 2002.

The loss of "chaotic" variability in the beat of the aging heart - constructing a delay difference equation

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Data from EKG shows that as people age their heart-beat looses variability. If we plot the RR interval (the time intervals between beats) as a function of beat number, the young healthy heart shows more variability than the old one, exhibiting an irregular pattern.
This "chaotic" dynamics of the young heart-beat represents physiologic vitality which is lost as the heart ages. The transition to periodic oscillations indicates a compromise of cardiac function. The goal is to construct models of difference equations for young and old hearts that will explain the data observed in the EKG; and ultimately to develop equations that will relate a particular beat to the previous ones. The equations will display changes in dynamical patterns based on parameter values.
The initial characterization of the system is carried out via statistical analysis in order to produce a phase map that will enable us to explore the order of the equations. Our preliminary results show some loss of non-linearity as the heart ages, perhaps indicating loss of physiologic regulation.

The rotation matrix for vertex maps on graphs

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It is well-known that for maps of the circle that are homotopic to the identity, the rotation interval is defined. This talk shows a way of extending this idea to certain maps on graphs.

Let G be a finite connected graph. Suppose $f\colon G \to G$ is a map homotopic to the identity that is also monotonic on the edges and permutes the vertices, then for such a map there is a *rotation matrix*, R, that is easily calculated. In this talk R will be calculated for some examples, and some basic properties of R will be described. In particular, in the case of the circle, it will be shown how the elements along the main diagonal of the matrix relate to the length of the rotation interval.

Perturbations of second order nonoscillatory scalar linear dynamic equations on time scales

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We are interested in the asymptotic behavior of solutions of

$$[r(t)x^{\Delta}]^{\Delta} + f(t)x^{\sigma} = 0, \qquad t \geq t_0, \tag{1}$$

as a perturbation of

$$[r(t)y^{\Delta}]^{\Delta} + g(t)y^{\sigma} = 0, \qquad t \geq t_0, \tag{2}$$

which is assumed to be nonoscillatory at infinity (here $r(t) > 0$).
The goal has been to identify conditions on the difference $f - g$ as well as the solutions of (2) so that solutions of (1) will be have asymptotically as those of (2) and to find estimates of the error terms.
This problem has drawn significant attention in the past. It was studied extensively in the setting of differential equations and to some extend for difference equations and time scales. Methods used include fixed point and Riccati techniques for scalar equations. Here we offer a new approach to this problem. Working in a matrix setting, we use preliminary and so-called conditioning transformations to bring the system in the form

$$\vec{z}^{\Delta} = [\Lambda(t) + R(t)]\vec{z},$$

where Λ is a diagonal matrix and R is a "small" perturbation. This allows us to apply Levinson's Fundamental Theorem on time scales to find the asymptotic behavior of solutions of (1) and to estimate the error involved.
This method allows us to derive new results and to improve existing results.

This is a joint work with Professor Donald A. Lutz from San Diego State University.

On equilibrium stability of difference equation in Banach space

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The paper deals with the mappings of Banach space E given in a form of quasilinear difference equation

$$x_{n+1} = Ax_n + F_n x_n,\ n \geq 0 \tag{1}$$

where A is linear continuous operator, $\{F_n \colon E \to E\}$ are nonlinear bounded operators satisfying identity $F_n 0 \equiv 0$ and having infinitesimal limit at equilibrium point 0:

$$\limsup_{\|x\| \to 0} \frac{\sup\limits_{n \geq 0} \|F_n x\|}{\|x\|} = 0$$

Side by side with the above equation we consider an equation of the first approximation, that is, the linear difference equation

$$y_{n+1} = Ay_n,\ n \geq 0 \tag{2}$$

We will discuss the assertions which guarantee local stability or instability for the trivial solution of (1) if (2) to be of this specificity. It should be mentioned that our paper not only generalizes well known finite dimensional results how it has been done in our previous papers (see, for example, [1]). Our research shows that the infinite dimension of the space E not only strongle complicates computations and proofs of relevant theorems by the first approximation but also can have significant influence to statement of these results (see, for example, [2],[3]).

[1] V.Yu. Slyusarchuk and Ye.F. Tsarkov (J.Carkovs), Difference equations in Banach space, *Latvian Mathematical Yearbook* No. 17, (1976), Riga, Zinatne, 214–229. (Rus.)

[2] V.Yu. Slyusarchuk, New theorem on instability of difference equations in linear approximation, *Scientific bulletin of Chelm, Section of mathematics and computer science* No. 1, (2007), 145–147.

[3] V.Yu. Slyusarchuk, *Equations with Essentially instable solutions*, Rivne, National University of Water Management and Natural Resources Application, (2005), 217 p. (Ukr.)

Extension of results guaranteeing a priori estimates of the boundary value problem solutions and their derivatives[1]

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For the ordinary second order non-linear differential equation two point boun- dary value problems in the monographs [1], [2] were formulated conditions, which ensure the a priori estimates of the problem solutions and their derivatives. The condition guaranteeing a priory estimate of the solution of such problems was signified in the well known terms of the lower and upper functions α and β. Of course, the generalizations of classical Nagumo's and Schrader's conditions ensure also a priori estimate for the derivative of the boundary value problem solution having a priori estimate. Moreover, such conditions are possible express in the terms of differential inequalities, which involves some functions realizing a priori estimate of the solution derivative, for example, ϕ and ψ. There were in the singular cases also used the one-sided estimates for the right part of equation.

We consider the possibility to carry out the analogous results for the corresponding boundary value problems for the difference equations as well as equations on time scales.

[1] I.T. Kiguradze, Some Singular Boundary Value Problems of the Ordinary Differential Equations, Tbilisi, 1975. (Russian).

[2] N.I. Vasiljev and Ju.A. Klokov, Foundations of the Boundary Value Problems Theory of the Ordinary Differential Equations, Riga, "Zinatne", 1978. (Russian).

[1]Supported by ESF research project 2009/0223/1DP/1.1.1.2.0/09/APIA/VIAA/008.

A periodically-forced mathematical model for the seasonal dynamics of malaria in mosquitoes

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We describe and analyze a periodically-forced difference equation model for malaria in mosquitoes. This model captures the effects of seasonality and allows the mosquitoes to feed on a heterogeneous population of hosts. We show that the model is mathematically and physically well-posed, and numerically show the existence of a unique globally asymptotically stable periodic orbit. We calculate field-measurable parameters that describe the level of malaria transmission in a given location and can be compared to data. We link this model with an individual-based stochastic simulation model for malaria in humans to compare the effects of different malaria control interventions in reducing malaria transmission, morbidity, and mortality.

Exact rates of decay of discrete Volterra convolution equations whose kernels have known periodic and decay components

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We consider a Volterra convolution summation equation where the kernel has specific decay and periodic components. By a careful splitting of the summation we isolate the periodic components and apply results on admissibility theory of Volterra operators to analyse the decaying component. In general, we show (roughly speaking) that if the kernel k decomposes according to $k(n) \sim p(n)\gamma(n)$ as $n \to \infty$ where p is an asymptotically N–periodic function, and γ is in a class of slowly decaying functions, then the solution $x(n)$ has asymptotic behaviour given by $x(n) \sim q(n)\gamma(n)$ as $n \to \infty$ where q is an asymptotically N–periodic function. This extends work of Appleby, Győri and Reynolds (2006), in which the kernel does not have a periodic component. We show a worked example of this theory relevant to the decay of the autocorrelations of an ARCH(∞) process.

This is a joint work between John Appleby and John Daniels.

[1] J. A. D. Appleby, I. Győri, and D. W. Reynolds, On exact convergence rates for solutions of linear systems of Volterra difference equations, *J. Difference Equations and Applications*, **12**, 12 (2006), 1257-1275.

The structure of a set of solutions of two-point BVP between lower and upper functions

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We consider a two-point nonlinear BVP of Dirichlet type provided that there exist well ordered lower and upper functions $\alpha(t)$ and $\beta(t)$ $(\alpha < \beta)$. We assume that there exists a solution $\xi(t)$ of the BVP and the type of $\xi(t)$ is not zero. We study properties of a set of solutions to the BVP and establish in particular, lower bound of a number of solutions. The types of solutions are discussed also.

Maximum principles for one of the components of solution vector for system of functional differential equations

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The method to compare only one component of the solution vector of linear functional differential systems, which does not require heavy sign restrictions on their coefficients, is proposed in this talk. Necessary and sufficient conditions of the positivity of elements in a corresponding row of Green's matrix are obtained in the form of theorems about differential inequalities. The main idea of our approach is to construct a first order functional differential equation for the nth component of the solution vector and then to use assertions about positivity of its Green's functions. This demonstrates the importance to study scalar equations written in a general operator form, where only properties of the operators and not their forms are assumed. It should be also noted that the sufficient conditions, obtained in this talk, cannot be improved in a corresponding sense and in many cases does not require any smallness of the interval $[0, \omega]$, where the system is considered.

Sturmain theory for symplectic difference systems

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We consider the first order $2n$-dimensional difference system

$$z_{k+1} = \mathcal{S}_k z_k \tag{S}$$

with symplectic matrix $\mathcal{S}$, i.e.,

$$\mathcal{S}_k^T \mathcal{J} \mathcal{S}_k = \mathcal{J}, \quad \mathcal{J} = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}.$$

System (S) covers as special cases a variety of difference equations and systems, among them also the Sturm-Liouville difference equation $\Delta(r_k \Delta x_k) + p_k x_{k+1} = 0$. We show that the classical Sturmian separation and comparison theorems can be extended in a natural way to (S). The presented results have been achieved in the joint research with Martin Bohner (University of Missouri, Rolla) and Werner Kratz (University of Ulm).

[1] M. Bohner, O. Došlý and W. Kratz, Sturmian and spectral theory for discrete symplectic systems, *Trans. Amer. Math. Soc.* **361**, 5 (2009), 3109–3123.

[2] O. Došlý and W. Kratz, Oscillation theorems for symplectic difference systems, *J. Difference Equ. Appl.* **13**, 7 (2007), 585–605.

[2] O. Došlý and W. Kratz, A Sturmian separation theorem for symplectic difference systems, *J. Math. Anal. Appl.* **325**,1 (2007), 333–341.

Molecular modeling of single and multiple β - sheets of amyloid B- protein ($A\beta$) 25 - 35[2]

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Molecular dynamics represents the computer approach to statistical mechanics, estimating equilibrium, stability and dynamic properties of a molecule system. The time-dependent behavior of the system is described by Newton's equations of motion, with the potential energy between two atoms described as:

$$U = U_{bond} + U_{angle} + U_{tors} + U_{vdW} + U_{elst} + U_{Hbond}, \tag{1}$$

where U_{bond} and U_{angle} are the bond and bond-angle distortion energies, U_{tors} is the torsional energy, U_{vdW} is the energy of van der Waals non-bounded interactions, U_{elst} is the electrostatic-interaction energy, and U_{Hbond} is the hydrogen-bonding energy. Equation (1) together with a set of parameters define a force field. Amyloid beta protein is responsible for formation of human amyloidosis leading to Alzheimer disease. A parallel single six stranded β-sheet of amyloid β-protein 25-35 (Aβ 25-35) built from the peptide Aβ 25-35 strands and the stack of six beta sheets of Aβ 25-35 were investigated by means of molecular dynamics (MD), Amber 9.0 force field, using isothermal-isobaric ensemble (NTP) protocol, chlorine counterions, explicit water molecules.

[2]This work was supported by European Economic Area block grant "Academic Research" LV0015. EEZ09AP-68 "Molecular modeling of amyloid formation", France-Latvia project "Osmose". Calculations were performed on computers of the Gdansk Academic Computer Center TASK.

Bifurcation and invariant manifolds of competition models

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In this paper we study two-dimensional competition models that arise in population dynamics. In particular, we investigate the invariant manifolds, including the stable and unstable manifolds as well as the important center manifolds. Saddle-node and period doubling bifurcation route to chaos is exhibited via analytic methods and numerical simulations. In addition we will employ Liapunov exponent techniques to study stability and chaos. Our models arise in the study of the distribution of mammals in some regions in North America and in the study of invasive grass species in Texas.

Joint work with R. Luis, M. Guzowska, H. Oliveira.

[1] M. Guzowska, R. Luis, and S. Elaydi, Bifurcation and invariant manifolds of the logistic competition model, *Journal of Difference Eq. Appl.* (2010), to appear

Coexistence of oscillations and nonoscillations in delay equations

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In this work we study the existence of both oscillatory and nonoscilatory solutions of delay equations. This analysis is made for difference equations

$$x\,(t) = \sum_{j=1}^{p} a_j\,(t)\,x\,(t - r_j) + f\,(t)$$

and also for diferrential difference equations

$$x'\,(t) = \sum_{j=1}^{p} a_j\,(t)\,x\,(t - r_j) + f\,(t)\,.$$

We shall provide some sufficient conditions which guarantee the coexistence of both type of solutions. Several examples which dwell upon the importance of our results are also illustrated.

[1] A. Andruch-Sobiło and M. Migda, Bounded solutions of third order nonlinear difference equations, *Rocky Mt. J. Math.* 36, 1 (2006), 23-34.

[2] Z. Došla and A. Kobza, Global asymptotic properties of third order difference equations, *Comput. Math. Appl.* 48, 1-2 (2004), 191-200.

Periodic orbits of the Kaldor-Kalecki trade cycle model with delay

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We prove in [3] the occurrence of periodic orbits in a version of the renowned Kaldor trade model. This is a delay differential system given by (1) describing interactions between the national gross product y and the capital stock k. The delay is a constant inherent to the specific economy. In this work we obtain a sequence of Hopf bifurcations.
In [2] a delay $T > 0$ is incorporated in the investment in the Kaldor model as a reflection of the Kalecki's hypothesis of gestation lag. The resulting equations, named *Kaldor-Kalecki* model by the authors, are the following:

$$\begin{aligned} \dot{y}(t) &= \alpha\left[I\big(y(t),k(t)\big) - S\big(y(t),k(t)\big)\right], \\ \dot{k}(t) &= I\big(y(t-T),k(t)\big) - \delta k(t). \end{aligned} \tag{1}$$

I and S are the investment and the savings functions, respectively, α is the adjustment coefficient in the goods market, usually referred as speed of adjustment, and δ is the depreciation rate of the capital stock. Delays are incorporated, since the investment depends on the income at the time investment is planned and on the capital stock at the time investment is finished.
This is a joint work with M.V.S. Frasson, S.H.J. Nicola and Plácido Z. Táboas.

[1] N. Kaldor, A model of the trade cycle, *Economic Journal* 50, 197 (1940), 78-92.

[2] A. Krawiec and M. Szydłowski, The Kaldor-Kalecki business cycle model, *Annals of Operations Research* 89 (1999), 89-100.

[3] M. V. S. Frasson, M. C. Gadotti, S. H. J. Nicola and P. Z. Táboas, Periodic orbits of the Kaldor-Kalecki trade cycle model, to appear.

Global bifurcation analysis of a biomedical dynamical system

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We study a quartic dynamical system which models the dynamics of the populations of predators and their prey that use the group defense strategy in a given biomedical system and which is a variation on the classical Lotka–Volterra system:

$$\begin{aligned} \dot{x} &= \ \ x((1-\lambda x)(\alpha x^2+\beta x+1)-y) \equiv P, \\ \dot{y} &= -y((\delta+\mu y)(\alpha x^2+\beta x+1)-x) \equiv Q, \end{aligned} \tag{1}$$

where $\alpha \geq 0$, $\delta > 0$, $\lambda > 0$, $\mu \geq 0$ and $\beta > -2\sqrt{\alpha}$ are parameters. Such a quartic dynamical model was studied earlier, for instance, in [1]. However, its qualitative analysis was incomplete, since the global bifurcations of limit cycles could not be studied properly by means of the methods and techniques which were used earlier in the qualitative theory of dynamicalsystems.
Together with (1), we will also consider an auxiliary system

$$\dot{x} = P - \gamma Q, \qquad \dot{y} = Q + \gamma P, \tag{2}$$

applying to these systems new bifurcation methods and geometric approaches developed in [2] and completing the qualitative analysis of system (1).

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Spectral problem for second order finite difference operator[3]

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We consider the discrete spectral problem $Ay = \mu y$ for finite difference operator on uniform grid $x_j = jh, Nh = L$ where h is the grid parameter, μ is the eigenvalue and A, y are correspondly the 3-diagonal matrix and column-vector of $N+1$ order (eigenvector) with elements $y_j, j = \overline{0, N}$.
The finite difference expression Ay can be represented in the following form [1]: $-2(y_1-y_0)/h^2+2\sigma_1 y_0/h,\ j=0;\ -(y_{j+1}-2y_j+y_{j-1})/h^2,\ j=\overline{1,N-1};\ -2(y_{N-1}-y_N)/h^2+2\sigma_2 y_N/h,\ j=N$, where σ_1, σ_2 are nonnegative parameters. This spectral problem is used in order to solve the 3^{th} kind boundary value problem of second order derivatives with respect to $x \in (0, L)$.
Using the scalar product of two vectors $[y^1, y^2] = h(\sum_{j=1}^{N-1} y_j^1 y_j^2 + 0.5(y_0^1 y_0^2 + y_N^1 y_N^2))$ one can prove, that the matrix A is symmetric and $[Ay, y] \geq 0$. The spectral problem has following solution: $y_j^n = C_n^{-1}(\frac{\sin(p_n h)}{h}\cos(p_n x_j) + \sigma_1 \sin(p_n x_j)),\ j = \overline{0,N},\ \mu_n = \frac{4}{h^2}\sin^2(p_n h/2)$, where p_n are the positive roots of the following transcendental equation: $\cot(p_n L) = \frac{\sin^2(p_n h) - h^2\sigma_1\sigma_2}{h(\sigma_1+\sigma_2)\sin(p_n h)}, n = \overline{1, N-1}$. The two last roots p_N, p_{N+1} can not be obtained from this transcendental equation.
Due to value of the parameter $Q = \frac{L\sigma_1\sigma_2}{\sigma_1+\sigma_2}$ we can obtain one $(Q < 1)$ or two $(Q \geq 1)$ roots from following new transcendental equation: $\coth(p_n L) = \frac{\sinh^2(p_n h)+h^2\sigma_1\sigma_2}{h(\sigma_1+\sigma_2)\sinh(p_n h)},\ n = \overline{N, N+1}$ and $\mu_n = \frac{4}{h^2}\cosh^2(p_n h/2)$, $y_j^n = C_n^{-1}(-1)^j(\frac{\sinh(p_n h)}{h}\cosh(p_n x_j) - \sigma_1 \sinh(p_n x_j))$. Then determine the constants $C_n^2 = [y^n, y^n]$ we have the orthonormed eigenvectors y^n, y^m for all $n, m = \overline{1, N+1}$.

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Disconjugacy of critical difference operators

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The critical difference operators are the non-negative operators that can be turned to negative ones by small negative perturbations of their coefficients. This concept has been introduced in [2] for difference operators of the second order and generalized in [1] for $2n$-order Sturm-Liouville difference operators. Lately, we have studied the one term operator $l(y)_k = \Delta^n(r_k \Delta^n y_k)$ and, using a structure of the solution space of the equation $l(y)_k = 0$, we have found a criterion of criticality of this operator. Our next goal is to use these one term critical operators to find new oscillation criteria for more complex difference operators.

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An algebraic context for difference operators

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After introducing the so called h–deformed Weyl–Heisenberg algebra and its standard representations we discuss some consequences of the defining (commutator) relations. Then we will study some ladders and quivers in that algebra. Two sets of polynomials will play a crucial role in the algebraic representation of the difference operator. In the last part we will study the $h \searrow 0$ transition to the continuous setting.

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Stochastically perturbed Ricker equations

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The Ricker model

$$x_{t+1} = x_t \exp(r - \gamma x_t), \;\; t = 0, 1, 2, \ldots, x_0 > 0$$

where x_t is the population density at time t, e^r is the growth rate at small densities and γ is an environmental parameter, is one of the most used models for single-species population dynamics. I will discuss some stochastic versions of the model with special emphasis on qualitative behavior: what features are retained when we go from deterministic to stochastic, what new features appear when the stochastic perturbations are "large". In particular, we will look at a population-dependent branching model which is derived in much the same way as Ricker's original equation.

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Newton problem solutions for 3D parabolic differential equation

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The parabolic differential equation with Newton boundary conditions for a parallelepiped is formulated as follows
$\frac{\partial}{\partial t}c\left(x,t\right) = D\sum_{n=1}^{3}\frac{\partial^2}{\partial x_n^2}c\left(x,t\right) + \sum_{n=1}^{3}v_n\frac{\partial}{\partial x_n}c\left(x,t\right) + f\left(c,t,x\right)$, where $x = (x_1, x_2, x_3)$, $0 < x_n < l_n, n = \overline{1,3}, t > 0$. The initial condition is $c(x,0) = c_0(x)$ and the boundary conditions are $\gamma_{n1}\left(c\left(x,t\right)\right)_{|x_n=0} + \gamma_{n2}\left(\frac{\partial}{\partial x_n}c\left(x,t\right)\right)_{|x_n=0} = \Phi_n\left(x\backslash\{x_n\},t\right)$,
$\gamma_{n3}\left(c\left(x,t\right)\right)_{|x_n=l_n} + \gamma_{n4}\left(\frac{\partial}{\partial x_n}c\left(x,t\right)\right)_{|x_n=l_n} = \Psi_n\left(x\backslash\{x_n\},t\right)$.
The general solution of the problem is known, but our purpose is to show solution existence conditions expressed with the new limitations as bilateral inequalities that are joining the velocities v_n to the constants from the boundary conditions γ_{nm}.
Green function approach is used for finding analytic solution of the Newton problem. Program code algorithm for numerical implementation of the solution is developed. Program code contains two parts where the first one is for finding eigenvalues from transcendental equation and the second is for numerical integration. Message Passing Interface is used for binding together more than one processor that provides a faster computation. On the base of test example the accuracy of numerical solution is estimated. It was shown that given new limitations are important because they exclude divergent solutions.

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On the global character of a rational second order difference equation

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This talk refers to the global character of solutions of the rational Difference Equation

$$x_{n+1} = \frac{a + x_{n-1}}{A + x_n x_{n-1}}, \ \ n = 0, 1, \ldots$$

with positive parameters and positive initial conditions.
(This is a joint work with E. Camouzis, E. Drymonis, and G. Ladas)

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Positive solutions of second order Emden Fowler difference equations

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In this paper we consider second order Emden Fowler difference equation

$$\Delta^2 y_n = \alpha p_n |y_{n+1}|^\sigma \, sign \, y_{n+1}, \tag{1}$$

where $\alpha \in \{-1, 1\}, \sigma \in \mathbb{R} \setminus \{0, 1\}$ and p_n is positive for all $n \in \mathbb{N}$. For the equation (1) the asymptotic representations of all positive solutions are established when the sequence p_n satisfies condition $\lim\limits_{n \to +\infty} \dfrac{n \Delta p_n}{p_n} = k$, $k \in R$. The results are illustrated on examples.

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Stability of a difference equation with two delays

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We study the stability problem of a difference equation

$$x_n = ax_{n-k} + bx_{n-m},$$

where k, m are delays. The cases of real and complex coefficients a, b are considered.

This is a joint work with V.Malygina and T. Khokhlova.

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Stability and dynamics of differential equations with distributed delays

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Using stability charts, we compare the stability properties of the zero solution of delay differential equations with distributed delays and associated equations with a discrete delay. The procedure allows us to locate the parameters when bifurcations occur. Indeed, we present numerical examples when delay distributions not only lead to increased stability region in the corresponding parameter space, but also generate complicated solutions after the loss of the linear stability.

Permanence induced by life-cycle resonances: the periodical cicada problem

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Periodical cicadas (*Magicicada* spp.) are known for their unusually long life cycle for insects and their prime periodicity of either 13 or 17 years. One of the explanations for the prime periodicity is that the prime periods are selected to prevent cicadas from resonating with predators with submultiple periods (e.g., see [1]). Based on this idea, Webb [2] constructed mathematical models and gave a numerical example that periodically oscillating predators with 2- or 3-year period eliminate nonprime number periodical cicadas. However, in Webb's model, the interaction between well-timed cicada-cohorts and their predators is ignored. In our study, we construct an age-structured model for dynamically interacting predator and prey populations and consider the problem of the predator-resonance hypothesis. Our main result shows that preys are not necessarily eliminated by predators with submultiple periods since invasion of preys is always facilitated by their well-timed cohorts. It is also shown that synchronized life-cycles between predator and prey populations can produce a permanent system, in which no cohorts are missing in both populations. This contrasts with the result that systems with asynchronous life-cycles always have a stable coexistence state where perfect periodicity is maintained in both populations. These results suggest that resonances with predators are not always deleterious to their preys.

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Molecular dynamics of amylin amyloid single and multiple β - sheets[4]

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Molecular mechanics describes the potential energy of an atom system as the sum of all pairwise interactions of individual atoms. The potential energy between two atoms U

$$U = U_{bond} + U_{angle} + U_{tors} + U_{vdW} + U_{elst} + U_{Hbond}, \tag{1}$$

where U_{bond} and U_{angle} are the bond and bond-angle distortion energies, respectively, U_{tors} is the torsional energy, U_{vdW} is the energy of van der Waals non-bonded interactions, U_{elst} is the electrostatic-interaction energy, and U_{Hbond} is an additional term to reproduce the hydrogen-bonding energy. The parameter set of (1) in composition with the equation (1) defines a force field. Molecular dynamics represents the computer approach to statistical mechanics, estimating equilibrium, stability and dynamic properties of a molecule system. The time-dependent behavior of the system is described by Newton's equations of motion:

$$M\ddot{X} = F(X) = -\nabla U(X).$$

[4]This work was supported by ESF project 2009/0197/1DP/1.1.1.2.0/09/APIA/VIAA/014, European Economic Area block grant "Academic Research" LV0015.EEZ09AP- 68 "Molecular modeling of amyloid formation". Calculations were performed on computers of the Gdansk Academic Computer Center TASK.

The bifurcation scenario of the periodic Ricker-type competition model

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In this talk we give some notes about a bifurcation of a periodic solution of the $p-$periodic nonautonomous Ricker-type difference equation given by

$$(x_{n+1}, y_{n+1}) = F_n(x_n, y_n), n \in \mathbb{Z}^+,$$

where

$$F_n(x, y) = (xe^{K_n - x - a_n y}, ye^{L_n - y - b_n x}),$$

and $K_n = K_{n \, mod \, p} > 0$, $L_n = L_{n \, mod \, p} > 0$, $a_n = a_{n \, mod \, p} \in (0, 1)$, and $b_n = b_{n \, mod \, p} \in (0, 1)$. A special attention will be given to studying attenuance and resonance in two dimensional systems.

*Joint work with Saber Elaydi and Henrique Oliveira

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On the rational difference equation with period-two coefficient

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We consider the non-autonomous rational difference equation

$$x_{n+1} = \frac{x_{n-1}}{p_n + x_n x_{n-1}}, \quad n = 0, 1, \dots$$

where the initial conditions x_{-1}, x_0 are nonnegative real numbers and (p_n) denotes the period-two sequence of positive values a and b. We prove that zero is the unique eqilibrium point and that for $a > 1$ and $b > 1$ it is globally asymptotically stable. We investigate also asymptotic behavior of solutions of the above equation.

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Three laws of chaos

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Sensitive dependence on initial conditions is a well known feature of chaotic dynamical systems. A butterfly flapping its wings in Hong Kong can cause a thunderstorm in Riga several days later. Nevertheless, the behavior of this butterfly should not change the long-term averages – how many thunderstorms per year will be observed in Riga in the next 500 years. However, it may happen that there is also sensitive dependence on parameters. In this case, bringing an additional butterfly to Hong Kong may essentially change those averages. Additionally, one cannot predict whether getting rid of such dangerous species as wing-flapping butterflies from Hong Kong will improve or deteriorate the climate in Riga.

Functional nodal method for 2D Helmholtz equation and truncation error analysis thereof

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We consider a functional nodal method and its application to finding the numerical solution of 2D Helmholtz equation [1] in Cartesian coordinates with Dirichlet boundary conditions:

$$u_{xx} + u_{yy} + \sigma^2 u = 0, \Omega \subset \mathbb{R}^2,$$

$$u(x, y) = f(x, y), \forall (x, y) \in \partial\Omega.$$

Helmholtz equation is used in many fields of mathematical physics, for instance, in acoustics, electrodynamics, theory of elasticity and non-linear advection diffusion.

The method represents an algorithm of separating the 2D Helmholtz equation into two 1D equations by averaging the flux in each of the directions x and y. Flux continuity is preserved. As the result we obtain two symmetric 7-point difference schemes, which consist of 3 points fixed in one of the directions, and 4 points fixed in the other direction. A truncation error analysis within the domain Ω is done and advantages of this method in comparison to other methods are shown.

The considered nodal method allows us to obtain separate difference schemes for each of the direction and preserve flux and solution continuity and it is a certain advantage over other existing methods [2].

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Bifurcation equations and cyclic permutations of periodic non autonomous systems

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In this note we analyze the symmetry, with respect to the order of composition of the maps, of degenerate bifurcation equations of periodic orbits in non autonomous systems. We prove that cyclic permutations in the order of compositions do not affect the solution of the bifurcation equations in the parameter space.

Joint work with Emma D'Aniello

On the nonautonomous difference equation $$x_{n+1} = A_n + \frac{x^p_{n-1}}{x^q_n}$$

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In this paper we study the asymptotic behavior and the periodicity of the positive solutions of the nonlinear difference equation

$$x_{n+1} = A_n + \frac{x^p_{n-1}}{x^q_n} \quad n = 0, 1, ...$$

where A_n is a positive bounded sequence, p, q are positive constants and $x_{-1}, x_0 \in (0, \infty)$.

Linear q-difference fractional-order dynamic systems with finite memory

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In the paper we deal with q-fractional difference control systems, evolving on $\mathbb{R}_+$, with the initialization by the additional function φ that is vanished on a time interval $[0,\varepsilon)$. In that way, starting with q-difference at point $t_0>0$, we get only finite number of values $\varphi\left(\frac{t_0}{q^k}\right)$, for $k\in\{0,\dots,l\}$, of initializing function φ. We call such set of values, stated as the extended vector, by l-memory. Hence a dynamical system is defined together with initializing points of time and length of a memory:

$$\begin{aligned}\Delta_q^{\alpha}x(t) &= A(qt)x(qt),\ t>t_0,\\ x(t) &= (\varphi u_a)(t),\ t\leq t_0,\\ y(t) &= C(t)x(t),\end{aligned}$$

where α is any rational number, $q\in(0,1)$, matrices $A(\cdot)\in\mathbb{R}^{n\times n}$, $C(\cdot)\in\mathbb{R}^{p\times n}$ have coefficients depending on $t\in\mathbb{R}_+$, $\varphi\colon\mathbb{R}\to\mathbb{R}^n$ and $u_a\colon\mathbb{R}\to\{0,1\}$, $a\in\mathbb{R}_+$ is function such that $u_a(t)=0$ for $t<a$ and $u_a(t)=1$ for $t\geq a$. For this system we construct the formula of the solution and discuss possible applications in linear control theory (such conditions as the observability and controllability in s-steps).

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[6]On leave of absence from Białystok University of Technology, Poland. Partially supported by R&D unit CIDMA and FTC through program Ciência 2007.

Robustness of hyperbolic solutions under parametric perturbations

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Autonomous difference equations feature distinguished solutions like constant or periodic ones. In this talk we investigate their behavior under *parametric perturbations*, where constant parameters in the discrete equation are replaced by time-varying sequences. The corresponding functional analytical approach turns out to be a quite fruitful contribution to the general theory of discrete nonautonomous dynamical systems (cf. the references [1–4]).

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On the global character of solutions of a system of rational difference equations

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We investigate the local stability of equilibrium points, boudedness nature of solution, periodic solutions and the attractivity of the periodic solutions.

Decoupling and simplifying of noninvertible difference equations[7]

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In Banach space $\mathbf{X} \times \mathbf{E}$ the system of difference equations

$$\begin{aligned} x(t+1) &= g(x(t)) + G(x(t), p(t)), \\ p(t+1) &= A(x(t))p(t) + \Phi(x(t), p(t)) \end{aligned}$$

is considered. Sufficient conditions under which there is an local Lipschitzian invariant manifold $u\colon \mathbf{X} \to \mathbf{E}$ are obtained. Using this result we find sufficient conditions of partial decoupling and simplifying of the system of noninvertible difference equations.

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On a system of two exponential type difference equations

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Our goal is to investigate the existence of the positive solutions, the existence of nonnegative equilibrium and the convergence of a positive solution to a nonnegative equilibrium of the system of difference equations

$$y_{n+1} = (1 - \sum_{j=0}^{k-1} z_{n-j})(1 - e^{-By_n}), z_{n+1} = (1 - \sum_{j=0}^{k-1} y_{n-j})(1 - e^{-Cz_n})$$

where B, C are positive numbers, $k \in \{2, 3, ...\}$, $n = 0, 1, \ldots$ and the initial values $y_{-k+1}, y_{-k+2}, \ldots, y_0$, $z_{-k+1}, z_{-k+2}, \ldots, z_0$ are positive numbers.

This is a joint paper with G. Stefanidou and G. Papaschinopoulos.

Asymptotically periodic solutions of Volterra systems of difference equations

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A Volterra system of difference equations of the form

$$x_s(n+1) = a_s(n) + b_s(n)x_s(n) + \sum_{p=1}^{r}\sum_{i=0}^{n} K_{sp}(n,i)x_p(i)$$

where $n \in N$, $a_s, b_s, x_s \colon N \to R$ and $K_{sp} \colon N \times N \to R$, $s = 1, 2, \ldots, r$ is studied. Sufficient conditions for the existence of asymptotically periodic solutions of this system are derived.

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On zero controllability of abstract evolution control equations

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Let X, U be complex Hilbert spaces, where U is finite-dimensional with dimension $r \geq 1$, and let A be infinitesimal generator of strongly continuous C_0-semigroups $S(t)$ in X [4]. Consider the abstract evolution control equation [4].

$$\dot{x}(t) = Ax(t) + Bu(t), x(0) = x_0, \ \ 0 \leq t < +\infty, \tag{1}$$

where $x(t), \ x_0 \in X, u(t) \in U$ is a control function, $B : U \to X$ is a linear possibly unbounded operator, $W \subset X \subset V$ are Hilbert spaces with continuous dense injections, $W = D(A)$ equipped with graphic norm, $V = W^*$, the operator B is a bounded operator from U to V (see more details in [5], [8].

Evolution equation (1) is a common mathematical model for distributed control systems. It provides the unified abstract approach for investigation of partial differential control systems governed by both boundary and distributed control, functional differential control systems [2], [3] integro-differential control systems and others kinds of distributed control systems.

The exact null-controllability problem can be formulated roughly as follows.

Given predefined time t_1 and initial state x_0, the goal is to find out whether there exists an admissible control $u(t)$ driving x_0 to the zero final state, provided that a control will be turned off after predefined time $t_2, t_2 \leq t_1$.

Necessary and sufficient conditions of exact null-controllability for linear evolution control equations with unbounded input operator are presented. They are obtained by transformation of exact null-controllability problem to linear infinite moment problem, which is defined as follows.

Given sequences $\{c_n, n = 1, 2 \ldots, \}$ and $\{x_n \in X, n = 1, 2 \ldots, \}$ find an element $g \in X$ such that

$$c_n = (x_n, g), n = 1, 2 \ldots, \tag{2}$$

where $(x, y), x, y \in X$ is the inner product of X.

The problem formulated above has a long history and many applications in many branches of the system control theory.

It is well-known, that if the sequence $\{x_n, n = 1, 2, \ldots, \}$ forms a Riesz basic in the closure of its linear span, then the linear moment problem (2)) has a solution if and only if $\sum_{n=1}^{\infty} |c_n|^2 < \infty$ and vice-versa [1], [7]. This well-known fact is one of main tools for the controllability analysis of partial hyperbolic control equations.

However the sequence $\{x_n, n = 1, 2, \ldots, \}$ doesn't need to be a Riesz basic for the solvability of linear moment problem.

In this talk we present the null-controllability of control evolution equations for the case when the sequence $\{x_n, n = 1, 2, \dots, \}$ doesn't form a Riesz basic in its closed linear span. Application to difference control system are considered.

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On the third order boundary value problems

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For the third order autonomous nonlinear differential equations estimations of the number of solutions to two and three point boundary value problem are given. Estimations are based on precise formulas for ratios of successive segments where solutions do not change sign.

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Multiplicity of solutions for discrete problems with double-well potentials

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In this talk we present some basic multiplicity results for a general class of nonlinear discrete problems with double-well potentials. Variational techniques are used to obtain the existence of saddle-point type critical points. Partial difference equations as well as problems involving discrete p-Laplacian are considered. Finally, we discuss the boundedness of solutions and the applicability of these results.

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On some symmetric difference equations

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We give a survey of old results regarding some classes of symmetric difference equations and present some new ones. One of these classes is the following

$$y_n = \frac{P_{2m+1}^{2m+1}(y_{n-k_1}^r, y_{n-k_2}^r, \ldots, y_{n-k_{2m+1}}^r)}{P_{2m}^{2m+1}(y_{n-k_1}^r, y_{n-k_2}^r, \ldots, y_{n-k_{2m+1}}^r)}, \qquad n \in \mathbb{N}_0,$$

where $r \in (0,1]$, $m \in \mathbb{N}$, $1 \le k_1 < k_2 < \cdots < k_{2m+1}$

$$P_{2m+1}^{2m+1}(x_1, x_2, \ldots, x_{2m+1}) = \sum_{r=1, r \text{ odd}}^{2m+1} \sum_{\{t_1,\ldots,t_r\} \subseteq S_{2m+1}, t_1<t_2<\cdots<t_r} x_{t_1} x_{t_2} \cdots x_{t_r},$$

and

$$P_{2m}^{2m+1}(x_1, x_2, \ldots, x_{2m+1}) = 1 + \sum_{r=2, r \text{ even}}^{2m} \sum_{\{t_1,\ldots,t_r\} \subset S_{2m+1}, t_1<t_2<\cdots<t_r} x_{t_1} x_{t_2} \cdots x_{t_r},$$

with $y_{-k_{2m+1}}, \ldots, y_{-1} \in (0, \infty)$

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Extension of discrete LQR problem to symplectic systems

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In this talk we consider a discrete linear-quadratic regulator problem in the setting of discrete symplectic systems (S). We derive minimal conditions which guarantee the solvability of this problem. The matrices appearing in these conditions have close connection to the focal point definition of conjoined bases of (S). We show that the optimal solution of this problem has a feedback form and that it is constructed from a generalized discrete Riccati equation. Several examples are provided illustrating this theory. The results of this paper extend the results obtained earlier by the authors for the special case of discrete linear Hamiltonian systems. We will also discuss a generalization of these results to time scales. The results were obtained jointly with Vera Zeidan from the Michigan State University.

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Sufficient number of integrals for the p^{th} order Lyness equation

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We present a sufficient number of explicit integrals for the Lyness equation of arbitrary order. We use the staircase method to construct integrals of a derivative equation of the Lyness equation. Closed-form expressions for the integrals are given based on a non-commutative Vieta expansion. The integrals of the Lyness equation follow directly from these integrals. Previously found integrals for the Lyness equation arise as special cases of our new set of integrals.

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Some remarks on matrix finite-difference equations

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The systems of finite-difference equations of arbitrary order were obtained in previous author's papers [1] and [2], related to solvability of elliptic boundary value problems in non-smooth domains. For the Laplacian in a plane sector with Dirichlet boundary condition on one angle side and the Neumann condition on other ones there arises 2×2 - system of first order of type

$$A(\lambda + 1) = B(\lambda)A(\lambda) \tag{1}$$

with given matrix $B(\lambda)$.
One discusses certain solving variants for (1).

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Developing and use of propagator difference scheme for solving of ADR equations

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The propagator method was developed in order to elaborate new effective tools for mathematical modelling. At first it was used for solve of ADR equation in geochemical processes [1]. Stability criterions by use of von Neumann criterion were obtained. It was proved, our propagator difference scheme has essentially weaker restrictions for the time step than the central difference scheme, so it allows to get faster computation results. Later the area of use for the propagator method was extended. It was used by modelling of temperature changes in process of electrochemical machining [2]. More essential is the possibility of solving the nonlinear problems with our propagator method. We developed the propagator difference scheme for the dissipative Murray's equation, who describes biomedical processes [3]. We get useful conditions for the convergence and stability.

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Rare periodic and chaotic attractors and bifurcation groups in typical nonlinear dynamical discrete models

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The problems of the global dynamics of nonlinear systems, described by discrete equations, are under consideration. The paper is a continuation of our publications on rare attractors (RA) and a method of complete bifurcation groups (MCBG) recently proposed by one of the authors [1, 2]. In this paper some rare periodic and chaotic attractors have been obtained for different typical nonlinear dynamical systems. As examples we discuss using the method of complete bifurcation group for logistical equations with square and cube nonlinearity [2], population models [3] and a problem of turbulent (vortex) flow [Ch. Skiadas, Von Karman Streets Chaotic Simulation, 2009]. In the presentation a new software Discrete-ABC is discussed as well.

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The new theory of nonlinear differential and difference equations based on bifurcation groups and its applications

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A new approach for the global bifurcation analysis for nonlinear difference equations, based on the ideas of Poincaré, Birkhoff and Andronov, is proposed. The main idea of the approach is a concept of complete bifurcation groups and periodic branch continuation along stable and unstable solutions, named by the author as a method of complete bifurcation groups (MCBG) [1-2]. The article is widely illustrated using difference nonlinear models with one-degree-of-freedom. Rare attractors in difference nonlinear models, e.g. [3], can be found using the same approaches as for nonlinear ODE [4].

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Asymptotic properties of solution of Volterra difference equations with periodic coefficients

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A Volterra difference equation of the form

$$x(n+1) = a(n) + b(n)x(n) + \sum_{i=0}^{n} K(n,i)x(i),$$

where $n \in N := \{0, 1, 2, \dots\}$, $a, b, x \colon N \to R$, $K \colon N \times N \to R$ is investigated. Here sequences a and K are not identically equal to zero, and $b \colon N \to R \setminus \{0\}$ is ω-periodic and $\prod_{k=0}^{\omega-1} b(k) = \beta$. Sufficient conditions for the existence of weighted asymptotically periodic solutions of this equation are presented. So, for any nonzero constant c, there exists a solution x of the considered equation such that

$$\frac{x(n)}{\beta^{\left[\frac{n-1}{\omega}\right]}} = u(n) + v(n)$$

with $u(n) := c \prod_{k=0}^{n^*} b(k)$ and $\lim_{n\to\infty} v(n) = 0$ where the function $[\,\cdot\,]$ is the greatest integer function and n^* is the remainder of dividing $n-1$ by ω.
The presented results improve and generalized the results presented in [2].

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On Weyl–Titchmarsh theory for dynamic symplectic systems on time scales [8] [9]

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We establish Weyl–Titchmarsh theory for dynamic symplectic systems on time scales. It is known that these systems cover 2nd order Sturm–Liouvile equations as well as linear Hamiltonian and symplectic systems. Hence our theory generalizes and unifies recently obtained results (by Bohner, Clark, Gesztesy, Hinton, Schneider, Shaw, Shi, Sun, etc.) on Weyl–Titchmarsh theory. Firstly, we consider a regular spectral problem. In the next part we focus on a singular spectral problem, we introduce Weyl–Titchmarsh M function, discuss its properties, and investigate necessary and sufficient conditions for the limit circle and limit point cases.

[8] *Key words and phrases.* Dynamic symplectic system; Time scales; Weyl–Titchamrsh M function; Regular spectral problem; Singular spectral problem; Limit point case; Limit circle case.
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